



Matrix Pencil Method applied to electromagnetics, smart metering and bioelectromagnetics phenomena

by

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Identity of the Laboratory



- Employees:
 - 198 Faculty members and researchers;
 - 27 Engineers, Technicians and Administrative;
 - 151 PhD students, postdocs or CSD.
- Budget:
 - The annual research budget is approximately 6.3 M€ of which 93% came from own resources (European projects, ANR, PIA, CPER, FUI, BPI contracts, Industrial, ...). Adding salaries, it amounts to almost 16 M€.
- Research achievements for last 5 years:
 - 2400 publications and 32 patents;
 - 19 European projects;
 - 58 national projects;
 - 60 industrial projects.



Structure of the Laboratory

INSTITUT PASCAL sciences de l'ingénierie et des systèmes

5 Transverse Programs



Structure of the Laboratory

5 branches (Scientific heart):

Mechanical Engineering, Civil Engineering, Industrial Engineering (M3G)
 65 academic staff (61 EC, 4 BIATSS)

Image, Systems, Perception and Robotics (ISPR)
 31 academic staff (24 EC, 2 C, 5 BIATSS)

- Process Engineering, Energetics and Bio systems (GePEB)
 27 academic staff (23 EC, 1 PR Emeritus, 3 BIATSS)
- Photonics, Waves and Nano materials (PHOTON) 41 academic staff (30 EC, 1 C, 3 Emeritus PR, 7 BIATSS)
- Image-Guided Therapy (TGI)61 academic staff (50 EC, 3C, 8 BIATSS)



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ElectroMagnetique Compatibility Group (EMC)

3 Professors - 4 Associate Professors 1 Associate Researcher - 1 Engineer - 5 PhD - 1 PostDoc

- Area of interest: development of numerical models for electromagnetic characterization. It is applied to complex systems, scenes and wave / structure interaction.
- Objectives:
 - Evaluation of electromagnetic couplings
 - Development of experimental methods for the generation of disruptive electromagnetic environment or noise reduction
 - Analysis of information transmission and communication
 - Electronic Systems Immunity



Theoretical background: Matrix Pencil Method

MPM algorithm in space domain per example

lacksquare Finite series of exponential model with damped factor $lpha_{
m i}$

$$y(z) = x(z) + n(z) = \sum_{i=1}^{M} R_i e^{S_i z} + n(z) ; \quad 0 \le z \le L \qquad S_i = \alpha_i + j\beta_i$$
$$y(kZ_e) = x(kZ_e) + n(kZ_e) \approx \sum_{i=1}^{M} R_i q_i^k + n(kZ_e)$$

$$Z_e = \Delta z$$
 and $q_i = e^{s_i Z_e}$ where $s_i = \alpha_i + j\beta_i$

Hankel matrix of N samples (structured matrix)

Matrix Pencil Method to estimate a function Matrix decomposition $\mathbf{Y}_1 = \mathbf{Z}_1 \cdot \mathbf{R} \cdot \mathbf{Z}_2; \qquad \mathbf{Y}_2 = \mathbf{Z}_1 \cdot \mathbf{R} \cdot \mathbf{Z}_0 \cdot \mathbf{Z}_2$ Van der Monde Matrices $\mathbf{Z}_{1} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_{1} & z_{2} & \cdots & z_{M} \\ \vdots & \vdots & \ddots & \vdots \\ z_{1}^{N-L-1} & z_{2}^{N-L-1} & \cdots & z_{M}^{N-L-1} \end{bmatrix} \begin{bmatrix} 1 & z_{1} & \cdots & z_{1}^{L-1} \\ 1 & z_{2} & \cdots & z_{2}^{L-1} \\ \vdots & \vdots & \cdots & \vdots \\ 1 & z_{M} & \cdots & z_{M}^{L-1} \end{bmatrix}_{M \times L}$

 $\mathbf{Z}_0 = \operatorname{diag} [z_1, z_2, \cdots, z_M]; \quad \mathbf{R} = \operatorname{diag} [\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_M]$

Use We construct eigenvalues problem det { $\mathbf{Y}_2 - \lambda \mathbf{Y}_1$ } = 0 or det { $\mathbf{Z}_1 \mathbf{R} \begin{bmatrix} \mathbf{Z}_0 - \lambda \mathbf{I}_M \end{bmatrix} \mathbf{Z}_2$ } = 0

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Singular Value Decomposition (SVD)

Any Matrix Y can be decomposed on singular values. $Y = U\Sigma V^{T} \qquad UU^{T} = I \\ VV^{T} = I$ Application to left and right Hankel matrices $Y_{1} = U\Sigma V_{1}^{T} \\ Y_{2} = U\Sigma V_{2}^{T}$ $\Sigma = diag \{\sigma_{1}, \sigma_{2}, \cdots, \sigma_{h}\}, \text{ with } h = \min \{N - L, L + 1\} \\ \sigma_{1} \ge \sigma_{2} \ge \cdots \ge \sigma_{M} \ge \sigma_{M+1} \ge \cdots \ge \sigma_{h}$

- M is the number of significant singular values
- We construct a filtered matrices

 $\mathbf{U}^{M} = \mathbf{U} (1: N - L, 1: M)$ $\mathbf{V}_{1}^{M} = \mathbf{V}_{1} (1: L, 1: M)$ $\mathbf{V}_{2}^{M} = \mathbf{V}_{2} (2: L + 1, 1: M)$ $\Sigma^{M} = \Sigma (1: M, 1: M)$

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Function signatures (Poles and Residues)

 $\begin{array}{l} \textbf{We extract} \left\{ z_i \right\}_{1 \leq i \leq M} \\ \text{matrix,} \\ det \left\{ \mathbf{V}_2^{\mathbf{M}} - \lambda \mathbf{V}_1^{\mathbf{M}} \right\} = \mathbf{0} \end{array}$

 \square Residues ${\it R}$ are derived by solving a Van Der Monde linear system,

$$\begin{bmatrix} y(1) \\ y(2) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \cdots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_M^{N-1} \end{bmatrix} \cdot \begin{bmatrix} \mathcal{R}_1 \\ \mathcal{R}_2 \\ \vdots \\ \mathcal{R}_M \end{bmatrix}$$

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High resolution parametric methods

signal filtré

0.25

Temps (s)

bruit

0.25

Temps (s)

Matrix Pencil: a complex exponential basis with mitigation consideration of a white Gaussian noise (measurement scenario)

0.3

0.3

0.35

0.35

Noise variance n(t) $\sigma = 0.1$

n

-1

-2 ⊾ 0

0.5

-0.5

'n

0.05

0.05

0.1

0.1

0.15

0.15

0.2

0.2

Reconstructed Signal $x(t) = 2\cos(40\pi t)e^{-10t} + n(t)$



0.5

0.5

0.45

0.45

0.4

0.4





High resolution parametric methods

Choosing of window size, position and the number of samples





Application to the case of the first Hankel function H_0 : Three basic functions (M=3)

High resolution parametric methods



Signature

$$\begin{split} s_1 &= -6.5817 + 1.6111\mathrm{i} \\ s_2 &= -0.0410 + 1.0015\mathrm{i} \\ s_3 &= -0.6509 + 1.0516\mathrm{i} \\ \Re_1 &= 0.2846 - 1.5600\mathrm{i} \\ \Re_2 &= 0.2678 - 0.2797\mathrm{i} \\ \Re_3 &= 0.4189 - 0.6490\mathrm{i} \end{split}$$







Obtaining the current distribution over the wire systems by:

Measurement: Difficulty to access especially in case of embedded wires (wall, ground).

- **Transmission lines theory:** Analytical model,
- Antenna theory: Non linear resolution.



Radiation from thin wire systems

Vector Potential and Green Function





Cylindrical coordinates

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EM field in cylindrical coordinates



Evaluation of Electromagnetic Fields

$$\vec{H} = H\varphi \, \vec{e}\varphi$$
$$\vec{E} = E\rho \vec{e}\rho + Ez \vec{e}z$$

Avec:
$$H\varphi = -\frac{1}{\mu} \frac{dAz}{d\rho}$$

 $E\rho = \frac{s}{\gamma^2} \frac{d^2Az}{d\rho dZ}$
 $Ez = \frac{s}{\gamma^2} \left(\frac{d^2Az}{dZ^2} - \gamma^2 Az\right)$

EM field in cylindrical coordinates

$$G_0 = \frac{e^{-\gamma R_0}}{R_0}$$



$$\begin{split} \vec{H}_{\phi} &= \frac{-1}{4\pi\rho\gamma} \left\{ \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I^{''} - \gamma^{2}I \right) e^{-\gamma R_{0}} dZ_{0} + \left(\left(I\gamma\cos\theta_{0} - I^{'} \right) e^{-\gamma R_{0}} \right]_{-\frac{L}{2}}^{\frac{L}{2}} \right) \vec{e}_{\phi} \\ \vec{E}_{\rho} &= \frac{\eta}{4\pi\rho\gamma^{2}} \left\{ \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I^{''} - \gamma^{2}I \right) e^{-\gamma R_{0}} dZ_{0} + \left(\left(\gamma I^{'}\cos\theta_{0} - I^{''} + I\gamma^{2}\sin^{2}\theta_{0} + I\frac{\gamma}{R_{0}}\sin^{2}\theta_{0} \right) e^{-\gamma R_{0}} \right]_{-\frac{L}{2}}^{\frac{L}{2}} \right\} \vec{e}_{\rho} \\ \vec{E}_{z} &= \frac{\eta}{4\pi\gamma} \left(\int_{-\frac{L}{2}}^{\frac{L}{2}} \left(I^{''} - \gamma^{2}I \right) G_{0} dZ_{0} + \left[I\cos\theta_{0}G_{0}^{'} + I^{'}G_{0} \right]_{-\frac{L}{2}}^{\frac{L}{2}} \right) \vec{e}_{z} \end{split}$$

integral parts

closed form part

Under the conditions of transmission line theory, the integral terms vanishes



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Radiation from thin wire systems



In the case of neglecting integral part

$$\overline{H_{\Phi}} = \frac{1}{4\pi\rho} \left[e^{-\gamma R_0} \left(I \cos \theta_0 - \frac{I'}{\gamma} \right) \right]_0^L \overrightarrow{e_{\Phi}}$$

$$\overline{E_{\rho}} = \frac{\eta_0}{4\pi\rho} \left[\frac{e^{-\gamma R_0}}{\gamma R_0} \left(I \sin^2 \theta_0 - \gamma R_0 \cos \theta_0 \left(I \cos \theta_0 - \frac{I'}{\gamma} \right) \right) \right]_0^L \overrightarrow{e_{\rho}}$$

$$\overline{E_z} = \frac{\eta_0 \gamma}{4\pi} \left[\frac{e^{-\gamma R_0}}{\gamma R_0} \left(\left(1 + \frac{1}{\gamma R_0} \right) I \cos \theta_0 - \frac{I'}{\gamma} \right) \right) \right]_0^L \overrightarrow{e_z}$$





Reconstruction of current distribution





Excellent agreement between Feko results and results reconstructed by MPM method

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Total and partial Magnetic Field

6 × 10⁻⁶

¥ 3





The integral part can be neglected



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Reconstruction of current distribution



Excellent agreement between Feko results and results reconstructed by MPM method for each branch F

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Total and partial Electrical Field







The integral part can be neglected



Total and partial Magnetic Field

Hx Hfx

IHx

15

10

1.4 × 10⁻⁴

1.2

0.8

0.2

-15

-10

0

z(m)

€ 0.6 0.4





The integral part can be neglected





Simulation results (Feko and our approach)







Excellent agreement between Feko results and results reconstructed by MPM method for each branch F

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$= \begin{bmatrix} 0.06 \\ 0.05 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.02 \\ 0.01 \\ 0.$

The integral part can be neglected





The integral part can be neglected



U-shape of two-wire line





Source of 1 V and 20 Mhz of frequency and 5 cm distance between conductors.

The observation points are defined as follows:

y varying between -1 m and 7 m, z varying between -2 and 7 m, with x = 1 m, we choose these region in which the radiated field is high enough.
Radiated EM field







Electric field estimated by our approach and by Feko *Error less than 4*%

Magnetic field estimated by our approach and by Feko *Error less than 4*%



Influence of the coupling on the EM field





Influence of the coupling on the magnetic field

Influence of the coupling on the electric field

The observation points P (x, y, z) are:

x = 20 cm, y = 50 cm, z between -2 m and 7 m with a step of 10 cm

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Experimental setup





Single-line parallel to a ground plane

Current and magnetic field



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Electric and magnetic field





Electric Field in frequency domain

Magnetic Field in frequency domain

Conclusions and on-going work



- Analytical model based on discontinuities (electrical and geometrical) currents and there two first derivatives,
- Short computing time to evaluate radiated near and far field,
- Simulation and measurement results agree satisfactory,
- Sensor less EM Field setup based only on discontinuities current measurements (PostDoc 2018).



Clermont Auvergne University Sungkyunkwan University



Efficient characterization of scattering objects in the frequency domain

PhD: Mahmoud KHODJET-KESBA (2014)



Introduction



Autonomous vehicles:



Collaboration between:

- Pascal Institute of Blaise Pascal University (France)
- Intelligent Systems Research Institute of Sungkyunkwan University (South Korea)



Natural complex resonances



The scattered signal in the time domain is composed of two parts:

impulsive component,

a damped oscillatory component.



Natural complex resonances



Case of conducting sphere:

The backscattered field by a sphere is given by:



The backscattered field from



a conductive sphere



The backscattered field from the sphere (radius of 30 cm)

Identification chronogram





Supervised classification



Definition:

- Supervised classification is based on a database of training data to automatically generate rules for grouping of individual data into a number of classes. We chose three methods to achieve this classification.
- □ The classification is done in two steps:
 - Learning step,
 - Test step.



Reconstructed and simulated signals

Poles $(\gamma_n - t_n j)$	Residus $(b_n(r))$
-2.0289 - 13.487 j	-0.0142 - 0.0192 j
-0.1252 - 9.8605 j	0.0179 - 0.0091 j
-0.6280 - 9.6984 j	0.0724 - 0.0101 j
-1.8427 - 6.4904 j	-0.1326 - 0.0019 j
-0.4937 - 4.7976 j	-0.0168 + 0.0375 j
0.0001 - 4.6683 j	0.0745 + 0.0045 j

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Simulation results



Classification of canonical objects of different sizes:



- In total, we have 24 objects to be classified:
- Wire : l= 1, 1.3, 1.5, 1.7, 2, 2.4, 2.5, 3, and radius r= 0.0025 m
- Sphere : r= 0.15, 0.2, 0.28, 0.3, 0.35, 0.4, 0.45, and 0.5 m
- Cylinder : l=0.4 and r=0.17, l=0.4 and r=0.23, l=0.5 and r=0.15, l=0.5
- and r=0.25, l=0.6 and r=0.2, l=0.7 and r=0.1, l=0.8 and r=0.19, and l=1 and r=0.15 m

Classification methods	Naïve				
methous	Bayes	K=1	K=3 K=5		SVM
Classification accuracy	100 %	100 %	91.67 %	91.67 %	100 %

Simulation results



Classification of complex objects with different SNR and different excitation angles:







Simulation results



Learning data:

- 8 angles: 5, 15, 25, 35, 45, 55, 65 et 75°
- 20 measurements with SNR of 10 dB for each angle
- In total: 4 x 8 x 20 = 640 learning data

Test data using only component E₀:

- 8 angles: 5, 15, 25, 35, 45, 55, 65 et 75°
- 7 values of SNR: 0, 5, 10, 15, 20, 25 et 30 dB for each angle
- 80 measurments for each SNR

SNR (dB) 0 5 10 15	20 2
	-31

In total, $4 \times 9 \times 7 \times 90 = 17020$ data to tot

SNR (dB)	0	5	10	15	20	25	30
K-NN (K=3)	71.33 %	80.74 %	93.83 %	96.48 %	97.66 %	98.52 %	99.30 %
NB	58.98 %	74.84 %	86.64 %	87.03 %	87 %	87.07 %	86.29 %
SVM	73.95 %	81.37 %	94.45 %	97.58 %	99.41 %	100 %	100 % 54

Conclusions and perspectives



- We have proposed in this part to identify scattering objects with a small number of signatures (singular values, poles, residues).
- The integration of these signatures in complex systems should help to model completely and with limited numerical effort.
- These signatures are enough to effectively classify three families of four canonical objects and complex objects.
- The use of all components backscattered field effectively increases the classification.
- Use both transmit polarizations would add new EM components in the data and make the classification more robust.



PhD: Hala Najmeddine (2009)



Objective of load monitoring:

- **Regulate** the energy consumption.
- **Determination** of the operation schedule of individual appliances and their energy consumption.
- Non intrusive: No intrusion into the energy consumer's property.
- Allows the identification by the matrix pencil method (in time domain) of the consumption of electrical loads without intervention on individual loads.



Non-Intrusive Appliances



Load Monitoring method



Experimental Results









$$S = \frac{j\omega}{2\pi} = 0 + 50j$$
$$R = \frac{E\sqrt{2}}{2r.j} \approx 0 - 7.7j$$

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Experimental Results





Conclusions



- In this work, we present our contribution to Non Intrusive Appliance Load Monitoring.
- □ The MP method shows several advantages:
 - Robustness against measurement noises
 - Less data required to identify the original signal by using reduced number of poles and residues.
- Different realistic case scenarios have been studied and the results arising from this analysis seem to be rather promising,
- Moreover, this method also provides active power, reactive power and harmonic power in close form.
- The proposed method has been successfully integrated into a platform of a smart meter.
- This procedure has been patented nationally and internationally.





Analysis of Electrophysiological activity by using MPM

PhD: Zied MIZOURI (2017)

PEPS: First Support of Exploratory Project (June 2015-June 2016)

¹Institut Pascal, CNRS-UMR 6602 ²Neuro-dol, Inserm U1107



Analysis of peripheral nervous activity

- 1. Recording electrical nervous activity,
- 2. Extraction of field potentials or action potentials,
- 3. Identification of those responses by using Matrix Pencil method.

Main goal: Recognition of the nociceptive response





- Goal: Have a better understanding of **pain message** and **its propagation** from the peripheral to the central nervous system,
- Data base: Electrostimulation of sciatic nerve of rat,
- Proposal: Stimulus artefact removal using Matrix Pencil Method to focus only on useful information.

Clinical context



Neuron: Action potential



Peripheral nerve: field potential (sum of individual neurons)



Clinical context



Action potential:

Existence of a gradient of concentrations and a gradient of charges. Existence of leakage channels which makes the nerve fibre membrane permeable during stimulation.



Pathways of pain



Between painful message and perception of pain, we go through an electrochemical cascade, which can be summarized in 4 steps:



Experimental setup





- Our data base is composed of signals recorded on the sciatic nerve of a rat,
- □ The electrodes used are invasive electrodes (to improve the selectivity of recorded signals: close to the source).
- These electrodes are directly connected to an amplifier equipped with a filter (high-pass and low-pass to eliminate unwanted frequencies).

□ Stimulations ranged from 20mV to 400 mV (20mV step) ,

Stimulus width from 200µs to 500µs.



- Our method propose to decompose stimulus artifact into two symmetrical [A] an [B] parts (Fig. 1). In the case where stimulus artifact and response are clearly separated (Fig. 2)., the duration between artefact peaks allows us to ascertain the duration of [A] an [B] parts.
- Under the hypothesis that [A] and [B] are symmetrical, the proposed method reconstructed the exponential part (N) of [B] from the corresponding exponential decay part (P) of [A] and predict its evolution with the series of M damped exponentials.

Numerical results



Case of stimulus artefact saturation:

- Symmetry is not respected,
- Only exponential parts will be treated,
- In this case, we reconstructed the late part of exponential

decay $[a_2]$ from the window $[a_1]$ (Fig. 3).



- Hypothesis validation and reconstruction quality was assessed with Normalized Mean Square Error (NRMSE). 71
Numerical results



Stimulus artefact removal



Fig.4 Example of stimulus artefact removal (response and artefact separated)

Fig.5 Example of stimulus artefact removal (case of saturation)

Main results



- Existing hardware methods causes distortion of neural response in the case where stimulus artefact overlap neuronal response,
- Implemented method remove successfully stimulus artefact,
- Decomposing stimulus artefact into two symmetrical part is very useful in stimulus artefact identification and suppression,
- Unlike existing software method (based on template substraction) the stimulus artefact is directly identified from the processed data which reduce residual signals after its removal,
- In the case of stimulus artefact saturation, symmetry is not respected; predicting the artefact evolution from early samples gave promising results,
- Predicting contaminated part of stimulus artefact using matrix pencil method can be a solution.

Recording nervous activity of a rat





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Extraction of action potentials





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Matrix Pencil method identification





Classification of action potentials





Results



- Existing hardware methods causes distortion of neural response on the case where stimulus artefact overlap neural response,
- Implemented method remove successfully stimulus artefact,
- Decomposing stimulus artefact into two symmetrical part is very useful in stimulus artefact identification and suppression,
- Unlike existing software method (based on template substruction) the stimulus artefact is directly identified from the processed data which reduce residues after its removal,
- In the case of stimulus artefact saturation, symmetry is not respected; prolonging the artefact from early samples gave promising results,
- Predicting contaminated part of stimulus artefact using matrix pencil method can be a solution.



Summary



- In this work, we present our contribution to analyze Electrophysiological Activities (ENG).
- The Matrix Pencil Method shows several advantages:
 - Robustness against measurement noises,
 - Predicting contaminated part of stimulus artefact,
 - Less data required to identify the original signal by using reduced number of poles and residues.
- Classification of action or field potentials.
- Expectation to extract nociceptive signal fibers Aδ and C.
- Extension to EEG diagrams.
- Correlation to Migraine patients.

Applications of Matrix Pencil Method

Evaluation of the Radiation from thin wire systems



- Identification of the current distribution (space domain).
- Objective: To evaluate the radiated EM field by using current and derivatives at geometric and electric discontinuities.
- Target Classification by using UWB radar
- Classification of fixed and moving targets under smart mobility (frequency domain).
- Objective: Complete the visual sensors (on board cameras) with Ultra Large Band Radar to classify obstacles more sufficiently.

Non Intrusive Load Monitoring Appliance

- Smart Metering : Identification of electric loads by using the load curve already existed at the power meter (time domain)
- Objective: Electric billing detail including relevant uses (lighting, heating and domestic hot water)





- Classification of action or field potentials (time domain).
- Objective: Have a better understanding of pain message and its propagation from the peripheral to the central nervous system.



Thank you for your attention! Any questions?





Appendices



Electromagnetic field in cylindrical coordinates

Avec:
$$G_{-1} = \int R_0 \ G_0 \ dR_0$$

 $G'_0 = \frac{\partial G_0}{\partial R_0}$
 $G'_{-1} = \frac{\partial G - 1}{\partial R_0}$
 $G''_{-1} = \frac{\partial^2 G - 1}{\partial R_0^2}$
 $I' = \frac{\partial I}{\partial Z_0}$

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potential vectors

$$\vec{A} = \frac{\mu}{4\pi} \sum_{h=1}^{M} R_h \int_0^L e^{S_h Z_0} G_0 \, dZ_0 \vec{e_z}$$
$$\vec{A_h} = \frac{\mu}{4\pi} R_h \int_0^L e^{S_h Z_0} G_0 \, dZ_0 \vec{e_z}$$
$$\vec{A} = \sum_{h=1}^{M} \vec{A_h}$$

Radiation from thin wire systems



Electromagnetic Fields series expressions:

$$\vec{H_{\phi}} = -\frac{1}{4\pi} \sum_{h=1}^{M} R_h \int_0^L e^{S_h Z_0} \sin \theta_0 \frac{\partial G_0}{\partial R_0} dZ_0 \vec{e_{\phi}}$$
$$\vec{E_{\rho}} = \frac{\eta}{4\pi\gamma} \sum_{h=1}^{M} R_h \left(S_h \int_0^L e^{S_h Z_0} \sin \theta_0 \frac{\partial G_0}{\partial R_0} dZ_0 - \left[e^{S_h Z_0} \sin \theta_0 \frac{\partial G_0}{\partial R_0} \right]_0^L \right) \vec{e_{\rho}}$$
$$\vec{E_z} = \frac{\eta}{4\pi\gamma} \sum_{h=1}^{M} R_h \left(\begin{array}{c} \left(S_h^2 - \gamma^2 \right) \int_0^L e^{S_h Z_0} G_0 dZ_0 \\ - \left[e^{S_h Z_0} \left(\cos \theta_0 \frac{\partial G_0}{\partial R_0} + S_h G_0 \right) \right]_0^L \right) \vec{e_z}$$

- It is important to stress out that these expressions are general.
- They are problem dependent in order to choose the corresponding Green function.